Journal of Applied Mechanics and Technical Physics, Vol. 38, No. 2, 1997

AN EXPERIMENTAL STUDY OF THE NONLINEAR EVOLUTION OF INSTABILITY WAVES ON A FLAT PLATE FOR MACH NUMBER M = 3

Yu. G. Ermolaev, A. D. Kosinov, and N. V. Semenov

UDC 532.526

The transition of the laminar form of flow in a boundary layer to a turbulent state is known to be a complicated process including the onset and evolution of disturbances of various kinds, their interaction with each other and the mean flow, and the formation of diverse vortex structures localized in space and time [1–3].

Until recently, theoretical [1, 3, 4] and experimental [5–10] studies of the origin of turbulence in a supersonic boundary layer were confined to the linear stage of generation of instability waves. Experiments were carried out both for natural and controlled disturbances artificially introduced into a supersonic boundary layer. The results for natural disturbances [5–7] are of a qualitative character. The studies of the stability of controlled disturbances show good agreement between theoretical and experimental data in the region of linear evolution of instability waves [3, 8–14].

Both experimental and theoretical research on the nonlinear instability of a supersonic boundary layer have been started recently [15–19]. The nonlinear development of disturbances has been studied in [17–19] by direct numerical simulation including the full Navier–Stokes equations. Erlebacher and Adams et al. [17, 19] have treated the temporal instability of a compressible boundary layer, and Eissler and H. Bestek [18] described a numerical experiment close to the conditions of a virtual experiment in studying the spatial development of disturbances. For natural disturbances, Kendall and Kimmel [15] have shown the existence of a nonlinear interaction of disturbances in the boundary layer of a 7-degree cone at M = 8.0. The nonlinear evolution of artificially excited wave packets in a supersonic boundary layer on a flat plate with a sharp leading edge at M = 2.0 were measured by Kosinov et al. [16]. The obtained wave characteristics of developing nonlinear disturbances showed that the mechanism of nonlinear wave interaction has a resonance character and is similar to the parametric-resonance mechanism in a subsonic boundary layer [20]. The subharmonic resonance in a boundary layer at M = 2.0 occurs for nonsymmetric wave triplets, whereas symmetric wave triplets are resonant in an incompressible boundary layer [16, 20, 21]. Note that the numerical results obtained by Adams and Sandham [19] confirm the qualitative and experimental data of [9, 10, 16] in the linear and nonlinear regions of evolution of instability waves.

It is known that the laminar–turbulent transition in a supersonic boundary layer depends on the Mach number [1, 3, 4]. Therefore, it seems important to carry out experiments, which are similar to [16], at different Mach numbers. Below, we present the results of an experimental study of the nonlinear development of instability waves in a supersonic boundary layer on a flat plate at M = 3.0.

1. Experimental Equipment. Experiments were performed in a T-325 supersonic wind tunnel at the Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences with a $200 \times 200 \times 600$-mm test section at M = 3.0 and at the unit Reynolds number $\mathrm{Re}_1 = U/\nu = 11.2 \cdot 10^6\ \mathrm{m}^{-1}$.

The experimental layout is shown in Fig. 1. The model was a flat steel plate, 450 mm long, 200 mm wide, and 10 mm thick. The angle of taper of the leading edge was $14°30'$, and the thickness of its blunt section was 0.02 mm. The model was mounted at zero incidence in the central plane of the test section. To introduce controlled oscillations into the boundary layer, a generator of localized disturbances was employed. In [21], Kosinov et al. described its design which is based on a spark discharge in the chamber. Controlled disturbances were introduced into the boundary layer through an aperture 0.42 mm in diameter in the working surface of the

Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 38, No. 2, pp. 107–114, March–April, 1997. Original article submitted October 30, 1995.

0021-8944/97/3802-0265 $18.00

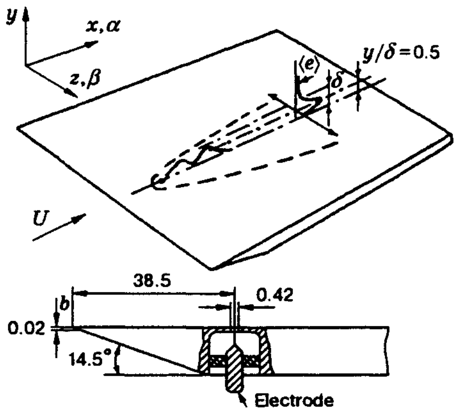

Fig. 1

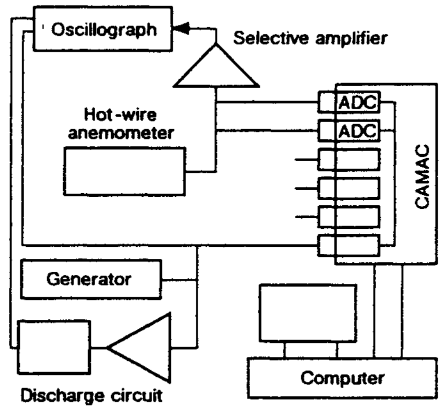

Fig. 2

model. The ignition frequency of the discharge was 20 kHz. The source coordinates were $x = 38 \pm 0.25$ mm and $z = 0$ (x is the distance from the leading edge of the model, and $z = 0$ corresponds to the model centerline).

To measure fluctuations, we employed a constant-temperature hot-wire anemometer and probes with a tungsten wire 5 μm in diameter and 1.2 mm long. The measurements were performed in the layer of maximum fluctuations across the boundary layer (along y/δ, where δ is the boundary-layer thickness). The hot wire was moved by a traversing gear in the direction of the x, y, and z axes. The accuracy of determination of the probe's position was 0.1 mm in the x and z axes and 0.01 mm along the y axis. In moving the probe along the x coordinate, the mean voltage in the diagonal of the hot-wire anemometer's bridge was kept constant owing to the displacement of the probe in the direction of the y axis, which is equivalent to measurements with $\rho u =$ const (ρu is the mass flow) and $y/\delta =$ const [5, 7, 9]. When the probe was moved along the z axis, the measurements were performed under the $x =$ const and $y =$ const conditions. The overheat ratio was 0.8, and the measured disturbances corresponded to the mass-flow fluctuations.

The pulsations and mean characteristics of the flow were measured using an automatized data acquisition system presented in Fig. 2. The fluctuating (within the frequency range up to 100 kHz) and mean voltages of the hot-wire anemometer were put into a computer through a 10-bit ADC with a frequency 1 MHz. The ADC was triggered simultaneously with the disturbance generator. The accuracy of ADC triggering was not worse than 0.2%. To improve the signal-to-noise ratio, a simultaneous summation of 500 realizations of the signal was performed directly during the experiment. The time of a realization was 200 μsec. The frequency harmonics were determined from the averaged oscillograms using a discrete Fourier transform (DFT). In the course of experiment, one of the harmonics obtained and the averaged fluctuating-signal oscillograms were displayed. This allowed the z boundaries of the introduced wave train to be found fairly accurately. For spectral processing of the experimental data, we used the DFT in the form

$$e_{\beta\omega}(x, y) = \frac{1}{T} \sum_{j,k} e(x, z_j, y, t_k) \exp\left(-i[\beta z_j - \omega t_k]\right),$$

where $e(x, y, z_j, t_k)$ is a fluctuating signal from the hot-wire anemometer and T is the time of one realization. After the DFT had been performed, the disturbance amplitude and phase were found by the formulas

$$A_{\beta\omega} = \{\mathrm{Re}^2[e_{\beta\omega}(x, y)] + \mathrm{Im}^2[e_{\beta\omega}(x, y)]\}^{0.5}, \quad \Psi_{\beta\omega} = \arctan\{\mathrm{Im}[e_{\beta\omega}(x, y)]/\mathrm{Re}[e_{\beta\omega}(x, y)]\}.$$

The flow parameters (M_∞, Re_1) were found based on the indications of a measurement system with which the facility was equipped. The pressure in the plenum chamber and the static pressure in the test section were measured by balance elements, and the stagnation temperature T_0 was measured by a thermocouple.

2. Results and Their Analysis. The experiments of [16] performed for a comparatively low level of initial disturbance amplitudes showed that in the boundary layer, for $M = 2$, the spectra of unstable disturbances in the nonlinear wave-generation region are three-dimensional. In this case, the main portion

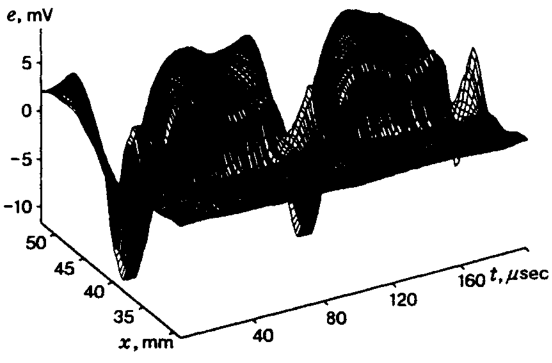

Fig. 3

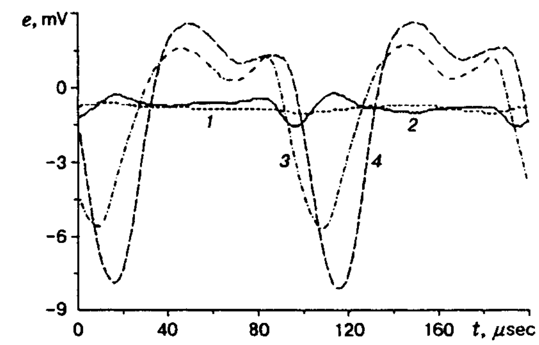

Fig. 4

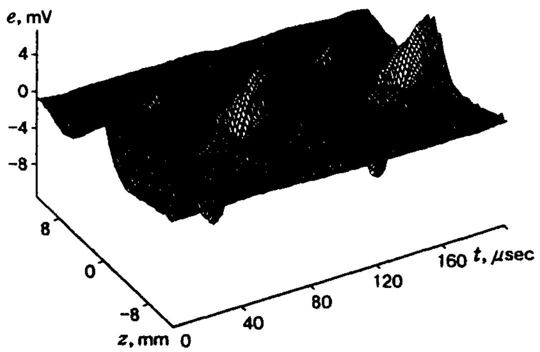

Fig. 5

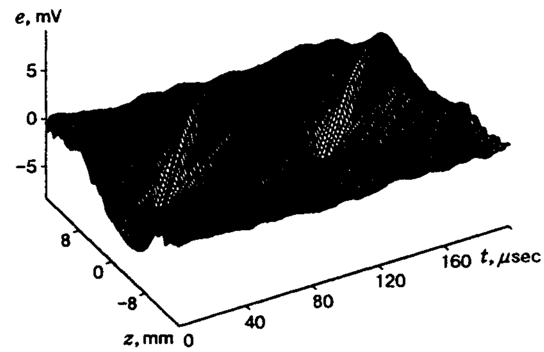

Fig. 6

of the energy of subharmonic disturbances is attributed to disturbances with the angles of inclination of the waves of approximately 80°. However, we have revealed that an increase in the initial disturbance amplitude changes qualitatively the nonlinear generation of wave trains. To study the features of the nonlinear stability of the supersonic boundary layer, we used the initial amplitude of controlled disturbances as a parameter that affects the nonlinear wave processes. The experiments described below were performed for the highest initial disturbance amplitudes which could be excited by the source of disturbances in use.

Initial Problem. The results of study of disturbances near the source are presented in Figs. 3 and 4. Figure 3 shows the three-dimensional surface of the averaged oscillograms of the pulsations $e(t)$ in the x direction for $z = 0$. Measurements in the x direction were performed with a step varied from 0.5 to 3 mm. These data show the evolution of the introduced disturbances from $x = 29$ mm. The disturbances therefore propagate not only downstream, but also upstream of the source (the coordinate of the source was $x = 38$ mm). While propagating upstream, the disturbances rapidly damp. This is seen in Fig. 4 which shows the pulsations $e(t)$ for various x. Curves 1 and 2 are the oscillograms of disturbances upstream of the source ($x = 29$ and 33 mm), and curves 3 and 4 are the oscillograms of disturbances downstream of the source ($x = 38.9$ and 41.1 mm). The disturbances are registered upstream of the source from $x = 33$ mm. The fluctuation amplitude is approximately an order of magnitude less than that near the source ($x = 38.9$ mm). Farther upstream of the source (curve 1, $x = 29$ mm), the disturbances are hardly seen. The results presented validate the previous experimental data on the upstream propagation of disturbances from a localized source at M = 2, 3, and 4 [13]. Downstream of the source (curves 3 and 4), the disturbances had a 100-μsec period in the boundary layer.

Nonlinear Development of Disturbances. Figures 5 and 6 show the three-dimensional surfaces of the averaged oscillograms of the pulsations $e(z,t)$ for $x = 60$ and 130 mm, respectively. For the initial cross section (Fig. 5, $x = 60$ mm), the signal has periodic spike-shaped minima with a period of 100 μsec. The wave packet is localized in the region symmetric about $z = 0$. In evolution of disturbances up to $x = 130$ mm (Fig. 6), the fluctuating signal $e(z,t)$ becomes practically sinusoidal in shape. The transformation of spikes into a sinusoid

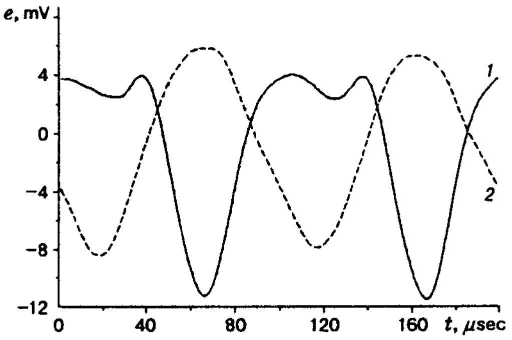

Fig. 7

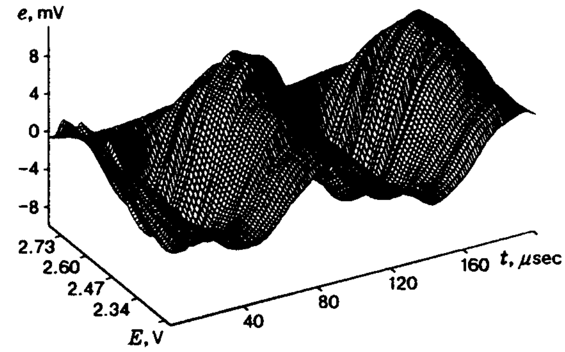

Fig. 8

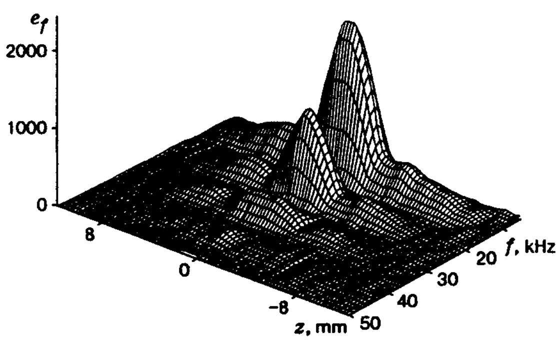

Fig. 9

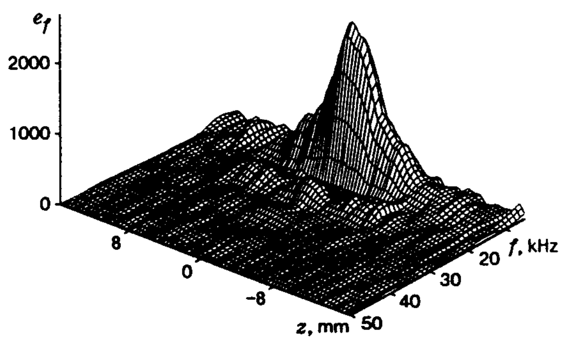

Fig. 10

is seen in Fig. 7 which shows oscillograms of disturbances with a maximum amplitude for $z \approx 0$. Curve 1 is the oscillogram for $x = 60$ mm and $z = 0$, and curve 2 is the same for $x = 130$ mm and $z = 0.1$ mm. Note that the oscillation amplitude remained practically unchanged from $x = 60$ to 130 mm.

To determine the character of changes in oscillations across the boundary layer, the fluctuations were measured in the y direction. The oscillograms obtained with the probe moving along the normal to the model surface for $x = 142$ mm and $z = 0$ are given in Fig. 8. Here E is the mean voltage of the hot-wire bridge. The increase in E is equivalent to the growth of the y coordinate. It is seen from Fig. 8 that the disturbances across the boundary layer have a maximum in the amplitude at $E = 2.54$ V. The fluctuating signal across the boundary layer has the same sinusoidal shape with a period of 100 μsec. Practically, no disturbances are traced in passing from the boundary layer in the free stream. This can be indicative of the vorticity nature of these disturbances which, as follows from the theory of hydrodynamic stability, are strongly attenuated outside the boundary layer.

For the data obtained to be treated within the framework of the stability theory, we have to analyze the wave-frequency results. Figures 9 and 10 show the amplitude-frequency spectra for $x = 60$ and 130 mm obtained by processing of the data presented above (see Figs. 5 and 6). Here e_f is the relative amplitude of disturbances at a fixed frequency. The amplitude-phase distributions along z were found for 10 frequencies with a step of 5 kHz. It is seen from Fig. 9 that the source generated disturbances at three basic frequencies (10, 20, and 30 kHz). The disturbances are localized in a narrow band at the center relative to z (±5 mm). As the wave packet developed downstream (Fig. 10) up to $x = 130$ mm, the disturbances with a frequency of 20 kHz decreased considerably in amplitude, and the wave packet degenerated at a frequency of 30 kHz. Smearing of the wave packet at a frequency of 10 kHz can be seen in Fig. 10. The full angle of smearing is 1.6°, which is substantially smaller than that predicted by the linear theory of stability (12°) [10, 14].

Figures 11–14 show wave-frequency spectra for $x = 60$, 130, 60, and 100 mm, respectively (the results of different-in-time experiments are considered). Here $e_{f\beta}$ is the relative disturbance amplitude. For the initial

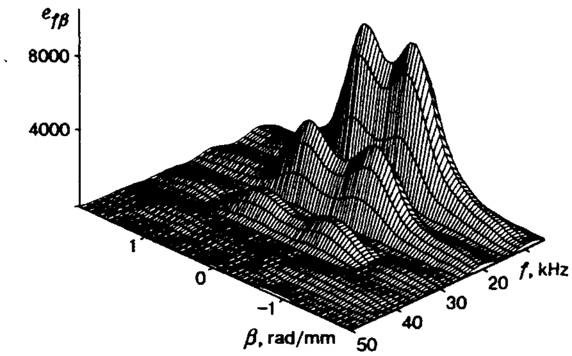

Fig. 11

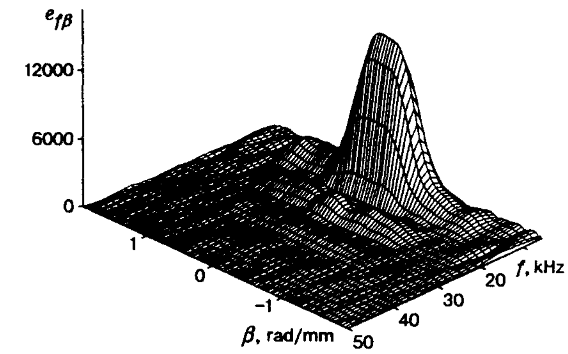

Fig. 12

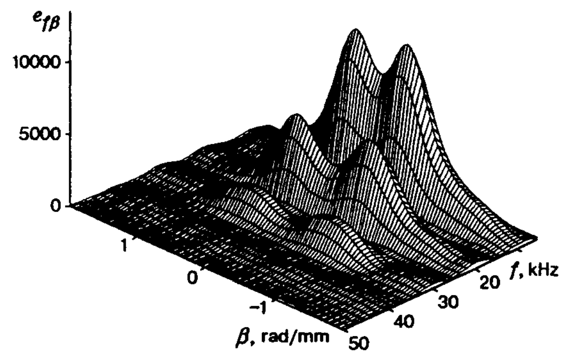

Fig. 13

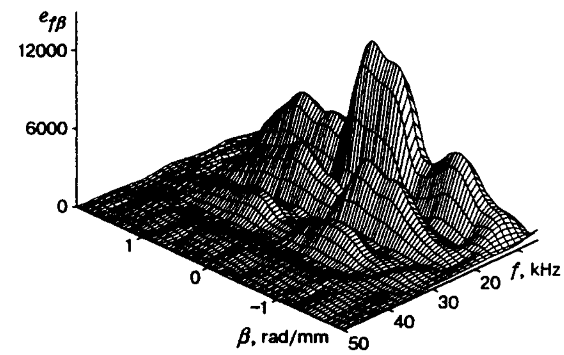

Fig. 14

cross section (Figs. 11 and 13), the amplitude spectra relative to β are typical of Tollmien–Schlichting waves which are linearly developed [10]. The data of Figs. 12 and 14 allow one to identify the wave processes that occur in the boundary layer as nonlinear processes: the spectra in Fig. 14 are substantially deformed, and the wave spectrum at a frequency of 10 kHz has peaks in the pronounced three-dimensional waves region of ($\beta \geqslant 1$ rad/mm). The latter fact is confirmed by the previous experimental studies of the nonlinear stability of the boundary layer at M = 2 [16].

Analyzing the above results, we should note that the appearance of rapidly increasing nearly two-dimensional ($\beta \approx 0$) (Figs. 12 and 14) and also three-dimensional ($|\beta| \geqslant 1$ rad/mm) (Fig. 14) waves is impossible to explain within the framework of the linear theory of stability.

However, from the viewpoint of the nonlinear concepts, the increments for two-dimensional and weakly inclined waves in the supersonic boundary layer can be assumed to be substantially higher than for strongly three-dimensional waves. Still, there is a need for an additional experimental investigation of these processes, a theoretical analysis of data with allowance for the contribution of the vortical and acoustic modes, and analysis of the role of the acoustic mode in nonlinear processes in the laminar–turbulent transition region of the supersonic boundary layer. If we take advantage of the analogy for the classical parametric resonance (amplification), we can expect that the parametric generation of acoustic disturbances is possible in a supersonic boundary layer after the parametric amplification of unstable waves. This scenario of nonlinear wave processes is indirectly validated by the data of [22] where the wave spectrum of generation of acoustic waves by the supersonic boundary layer is shown to have a maximum near $\beta = 0$.

This work was partially supported by the International Science Foundation (Grant No. NQZ000).

REFERENCES

1. S. A. Gaponov and A. A. Maslov, *Development of Disturbances in Compressible Flows* [in Russian], Nauka, Novosibirsk (1980).
2. Yu. S. Kachanov, V. V. Kozlov, and V. Ya. Levchenko, *Initiation of Turbulence in a Boundary Layer* [in Russian], Nauka, Novosibirsk (1982).
3. V. N, Zhigulev and A. M. Tumin, *Initiation of Turbulence* [in Russian], Nauka, Novosibirsk (1987).
4. L. M. Mack, "Boundary layer stability theory," in: Document 900-277, California-JPL, Pasadena (1969).
5. J. Laufer and T. Vrebalovich, "Stability and transition of a supersonic boundary layer on a plate," *J. Fluid Mech.*, **9**, 257–299 (1960).
6. J. M. Kendall, "Wind tunnel experiments relating to supersonic and hypersonic boundary-layer transition," *AIAA J.*, **13**, No. 3, 290–299 (1975).
7. V. A. Lebiga, A. A. Maslov, and V. G. Pridanov, "Experimental investigation of the stability of supersonic boundary layer on a flat insulated plate," *Arch. Mech.*, **31**, No. 3, 397–405 (1979).
8. J. M. Kendall, "Supersonic boundary-layer stability experiments," *Aerospace Rep. Tr.*, **2**, No. 158 (S3816-63)-1, 10-1–10-8 (1967).
9. A. D. Kosinov and A. A. Maslov, "Development of artificially induced disturbances in a supersonic boundary layer," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 37–43 (1984).
10. A. D. Kosinov, A. A. Maslov, and S. G. Shevel'kov, "Experiments on the stability of supersonic boundary layer," *J. Fluid Mech.*, **219**, 621–633 (1990).
11. A. D. Kosinov, A. A. Maslov, and S. G. Shevel'kov, "Experimental simulation of wave processes in a supersonic boundary layer on a flat plate," in: *Simulation in Mechanics* [in Russian], Inst. of Theor. and Appl. Mech., Sib. Div., Russ. Acad. of Sci., **7(24)**, No. 4 (1993), pp. 100–109.
12. A. D. Kosinov, A. A. Maslov, and S. G. Shevel'kov, "Experimental investigation of development of harmonic disturbances in the boundary layer of a flat plate for the Mach number M = 4," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 54–58 (1990).
13. A. D. Kosinov, N. V. Semionov, and S. G. Shevelkov, *Special Features of Generation and Development of a Harmonic Wave in a Supersonic Boundary Layer*, Int. Academic Press, Beijing (1992), pp. 53–58.
14. P. Balakumar and M. R. Malik, "Discrete models and continuous spectra in supersonic boundary layers," *J. Fluid Mech.*, **239**, 631–656 (1992).
15. J. M. Kendall and R. L. Kimmel, "Nonlinear disturbances in a hypersonic laminar boundary layer," AIAA Paper No. 91-0320, New York (1991).
16. A. D. Kosinov, N. V. Semionov, S. G. Shevelkov, and O. I. Zinin, "Experiments on the nonlinear instability of supersonic boundary layers," in: IUTAM Symp., Potsdam, New York (1994), 196–205.
17. G. Erlebacher and M. Y. Hussaini, "Numerical experiments in supersonic boundary-layer stability," *Phys. Fluids*, **2**, 94–104 (1990).
18. W. Eissler and H. Bestek, "Spatial numerical simulations of nonlinear transition phenomena in supersonic boundary layers," in: ASME Fluids Engineering Conference on Transitional and Turbulent Compressible Flows, June 21–24, Washington (1993).
19. N. A. Adams and N. D. Sandham, "Numerical simulation of boundary layer transition at Mach 2," *Appl. Sci. Res.*, **51**, 371–375 (1993).
20. Yu. S. Kachanov and V. Ya. Levchenko, "The resonance interaction of disturbances at laminar–turbulent transition in a boundary layer," *J. Fluid Mech.*, **138**, 209–247 (1984).
21. A. D. Kosinov, N. V. Semionov, and G. S. Shevelkov, "Investigation of supersonic boundary-layer stability and transition using controlled disturbances," in: *ICMAR Conf.*, Novosibirsk (1994), Part 2, pp. 159–166.
22. A. A. Maslov and N. V. Semenov, "Generation of acoustic oscillations by a supersonic boundary layer," *Izv. Akad. Nauk SSSR, Ser. Tekh. Nauk*, No. 7, Issue 2, 58–63 (1987).